Periodical Preventive Maintenance Schedule for Minimum Cost Using Geometric Process Model in Accelerated life Testing With Exponentiated Weibull Failure Data

Eman A. El-Dessouky

Department of Statistics
Faculty of Commerce
Al- Azhar University, Egypt.

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Abstract

Maintenance may be considered the heath care of our manufacturing machines
and equipment. It is required to effectively reduce waste and run an efficient, continuous manufacturing operation, business, or service operation. The planned maintenance of equipment will help to improve equipment life. The goal of this paper is to estimate the Periodical Preventive Maintenance Schedule which minimize the total cost of maintenance. The research concern on estimating the parameters of time failures Exponentiated Weibull distribution using Geometric process model in accelerated life testing . Maximum likelihood method is used to estimate the model parameters and acceleration factor of lifetime distribution from the test data. Confidence Interval for the model parameters are constructed using normal approximation and bootstrap method. we used the estimated parameters to obtain periodical preventive maintenance schedule which minimizes the total maintenance cost. Finally, Some numerical illustrations are provided .

Key words

Preventive Maintenance, Geometric Process Model, Accelerated life Testing
, Exponentiated Weibull distribution

Periodical Preventive Maintenance Schedule for Minimum Cost

1- Introduction

Preventive maintenance is planned maintenance of plant and equipment that is designed to improve equipment life and avoid any unplanned maintenance activity. It may be performed on a few selected components when a component fails. Importance measures can be used to identify the most important component that requires maintenance. There are many important benefits of preventive maintenance program such as: equipment downtime is decreased and the number of major repairs are reduced, reduced overtime costs and more economical use of maintenance workers due to working on a scheduled basis instead of a crash basis to repair breakdowns, and Improved safety and quality conditions for everyone. Many researches interested with preventive maintenance such as:

Kamran et al. (2010), presented a new mathematical function to model an improvement factor based on the ratio of maintenance and repair costs, and show how it outperforms fixed improvement factor models by analyzing the effectiveness in terms of cost and reliability of the system. Jain (2012), presented optimal preventive maintenance strategy for efficient Operation of boilers in industry. Siswanto and Kurniati, (2017), attempted to analyze the failure data and reliability based on historical data. Optimal preventive maintenance interval is determined in order to minimize the total cost of maintenance per unit time. Belyi et al. (2017), presented the optimal preventive maintenance schedule of a single item over a finite horizon, based on Bayesian models of a failure rate function. Iskandara and Husniah (2017), studied a two dimensional lease equipment contract with involving imperfect preventive maintenance by considering that an imperfect preventive maintenance (PM) policy reduces the equipment failure rate and hence it will decrease the penalty cost and maintenance cost during the lease contract. Park et al. (2018), considered an optimal periodic preventive maintenance policy after the expiration of two-dimensional warranty, and developed an optimal post-warranty periodic preventive maintenance strategy by minimizing the expected cost rate incurred during the life cycle of the system. Sgarbossa et al. (2018), studied the Impacts of weibull parameters estimation on preventive maintenance cost, the aim of this study is to investigate the relationship between the error in parameter estimation and the additional costs related to this error.

The rest of this paper can be organized as follows: In Section 2 our model is presented and its assumptions and test procedure for the geometric process model where life time distribution is Exponentiated Weibull . The maximum likelihood estimates of the model parameters are obtained in section 3. Section 4 presents The confidence interval for model parameters are constructed by using normal approximation method and bootstrap method. The periodical preventive maintenance schedual which minimizes the total maintenance cost is obtained in section 5 To illustrate the theoretical results, simulation studies are carried out in Section 6. Finally, Section 7contains the conclusions.

2. The Model

This section introduces the assumed model for product life

Notations:

x	lifetime of an item
λ	acceleration factor $\lambda > 1$
β	The scale parameter of the Exponentiated Weibull distribution
α, γ	the shape parameters of the Exponentiated Weibull distribution
t_0	The periodical preventive maintenance schedule
T	The total operating time of a component.
C_P	The cost of replacement of a component in the preventive maintenance
C_r	The cost of replacing of a component through corrective maintenance

2.1 Failure Time distribution as Exponentiated Weibull distribution:

The lifetimes of the test items are assumed to follow a three - parameter Exponentiated Weibull distribution.Mudholkar and Srivastava, 1993 offered generalization of the Weibull distribution as Exponentiated Weibull for modeling bathtub failure rate lifetime data. Jiang and Murthy (1999), a parametric characterization of the pdf and the failure rate for the exponentiated Weibull family are carried out. There are may studies of the EW distribution as the following: U. Singhet al. (2005) presented MEL and Bayesian estimation for the EW parameters. Lianfen Qian, (2012), illustrated graphically the shape property of the hazard function of EW and proposed algorithm for computing the maximum likelihood estimator and derives the Fisher information matrix. Mashail and Soliman, (2016), presented MLE and Bayes estimation for the exponentiated Weibull model with adaptive Type-II progressive censored schemes.

So, it is a statistical distribution frequently used in life data analysis. The probability density function (pdf) of three-parameter Exponentiated Weibull distribution is given by:

$$f(x) = \alpha\gamma\beta^{\gamma}x^{\gamma-1}e^{-(\beta x)^{\gamma}}(1-e^{-(\beta x)^{\gamma}})^{\alpha-1} \qquad ;x \geq 0,\ \ \alpha,\beta,\gamma > 0 \tag{1}$$

The cumulative function is given by:

$$F(x) = (1-e^{-(\beta x)^{\gamma}})^{\alpha} \qquad ;x \geq 0,\ \ \alpha,\beta,\gamma > 0 \tag{2}$$

The reliability function is

$$R(x) = 1-(1-e^{-(\beta x)^{\gamma}})^{\alpha} \qquad ;x \geq 0,\ \ \alpha,\beta,\gamma > 0 \tag{3}$$

The hazard function is

$$h(t) = \frac{\alpha\gamma\beta^{\gamma}x^{\gamma-1}e^{-(\beta x)^{\gamma}}(1-e^{-(\beta x)^{\gamma}})^{\alpha-1}}{1-(1-e^{-(\beta x)^{\gamma}})^{\alpha}} \qquad ;x \geq 0,\ \ \alpha,\beta,\gamma > 0 \tag{4}$$

2.2 The Geometric Process Model (GP)

The GP supposes that failure times $X_1, X_2, \ldots, X_n$ is a sequence of random variables, such that $\{\lambda^{n-1}X_n,\ n = 1,2,\ldots\}$ forms a renewal process where $\lambda > 1$ is real valued and called the ratio of the GP. Accelerated testing, lifetime will be stochastically
decreasing with respect to increasing stress levels. Therefore, the geometric process model is a natural approach to study such a problem. Suppose$\{X_k,\ k = 1,2,\ldots,s\}$ are lifetimes under each stress level . Many researchers Using Geometric Process in accelerated life testing For various probability distributions such as:
Huang (2011) exponential distribution and Weibull distribution used in M. Kamal et al. (2012) for complete data and M. Kamal (2013), with Type –I censored. M. Kamal et al. (2013) for Pareto distribution.

Saxena et al. (2013) Rayleigh distribution. Sana and Arif (2013) Marshall-Olkin Lomax Distribution and Frechiet distribution. Sadia et al. (2014), Marshall-Olkin Extended Exponential Distribution with type I censored data. Showkat et al. (2016) for generalized exponential distribution using time constraint. Kamal Ullah et al. (2017), generalized Rayleigh distribution. Mohie El-Din et al. (2018), for Extension of the Exponential Distribution.

2.3 Assumptions and test procedure

i. Suppose that an accelerated life test with *s* increasing stress levels in which a random sample of *n* identical items is placed under each stress level and start to operate at the same time. Let $x_{ik}, i = 1,2,\ldots,n \ , k = 1,2,\ldots,s$ denote observed failure time of i^{th} test item under k^{th} stress level. Whenever an item fails, it will be removed from the test and the test is terminated when all items in the test are failed.

ii. The product life follows Exponentiated Weibull distribution given by (1) at any stress.

iii. The scale parameter β is a log-linear function of stress. That is $\log(\beta_k) = a + b\, S_k$, where a and b are unknown parameters depending on the nature of the product and the test method , therefore $\beta_k = \lambda^k \beta$.

iv. The EW shape parameter γ is constant, therefore γ independent of stress.

Therefore the pdf of the product lie time at the k th stress level is:

$$\begin{aligned} f_{x_k}(x) &= \alpha\gamma(\lambda^k\beta)^\gamma x^{\gamma-1} e^{-(\lambda^k \beta x)^\gamma}\left(1 - e^{-(\lambda^k \beta x)^\gamma}\right)^{\alpha-1} \quad ; x \geq 0, \ \alpha,\beta,\gamma,\lambda > 0 \\ &= \lambda^k \alpha\gamma\beta^\gamma(\lambda^k x)^{\gamma-1} e^{-(\beta\lambda^k x)^\gamma}(1 - e^{-(\beta\lambda^k x)^\gamma})^{\alpha-1} \end{aligned} \tag{5}$$

Which means the life times of EW represent the sequence of geometric process and satisfy the condition: $f_{x_k}(x) = \lambda^k f(\lambda^k x) , k = 0,1,2,\ldots,s$. therefor life times under sequence of arithmetic increasing stress levels form a geometric process with ratio λ .

3. Maximum Likelihood Estimation

The likelihood function for the EW under geometric process model based on the observed data in a total stress levels in accelerated life testing, can be written as:

$$L(\underline{x}|\alpha,\beta,\gamma,\lambda) = \prod_{k=1}^{s}\prod_{i=1}^{n} \lambda^{k\gamma}\alpha\gamma\beta^{\gamma}x_{ki}^{\gamma-1}xx_{ki}{}^{\gamma-1}e^{-(\lambda^{k}\,\beta x_{ki})^{\gamma}}\left(1-e^{-(\lambda^{k}\,\beta x_{ki})^{\gamma}}\right)^{\alpha-1} \tag{6}$$

It is usually easier to maximize the natural logarithm of the likelihood function rather than the likelihood function itself. We can get the natural logarithm of the likelihood function as follows:

$$lnL(\underline{x}|\alpha,\beta,\gamma,\lambda) = \sum_{k=1}^{s}\sum_{i=1}^{n}\begin{bmatrix} k\gamma ln\lambda + ln\alpha + ln\gamma + \gamma ln\beta + (\gamma-1)lnx_{ki} \\ -(\lambda^{k}\,\beta x_{ki})^{\gamma} + (\alpha-1)ln\left(1-e^{-(\lambda^{k}\,\beta x_{ki})^{\gamma}}\right)\end{bmatrix} \tag{7}$$

The first derivatives of the natural logarithm of the total likelihood function in (7) with respect to $\lambda\,,\alpha\ and\ \beta$ are given by:

$$\frac{\partial lnL}{\partial\lambda} = \sum_{k=1}^{s}\sum_{i=1}^{n}\begin{bmatrix} \frac{k\gamma}{\lambda} - k\gamma\lambda^{k\gamma-1}(\,\beta x_{ki})^{\gamma} \\ + \frac{(\alpha-1)e^{-(\lambda^{k}\,\beta x_{ki})^{\gamma}}k\gamma\lambda^{k\gamma-1}(\,\beta x_{ki})^{\gamma}}{\left(1-e^{-(\lambda^{k}\,\beta x_{ki})^{\gamma}}\right)}\end{bmatrix} \tag{8}$$

$$\frac{\partial lnL}{\partial\alpha} = \sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{1}{\alpha} + ln\left(1-e^{-(\lambda^{k}\,\beta x_{ki})^{\gamma}}\right)\right] \tag{9}$$

$$\frac{\partial lnL}{\partial\beta} = \sum_{k=1}^{s}\sum_{i=1}^{n}\begin{bmatrix} \frac{\gamma}{\beta} - \gamma\beta^{\gamma-1}\lambda^{k\gamma}x_{ki}^{\gamma} \\ + \frac{(\alpha-1)e^{-(\lambda^{k}\,\beta x_{ki})^{\gamma}}\gamma\,\beta^{\gamma-1}\lambda^{k\gamma}x_{ki}^{\gamma}}{\left(1-e^{-(\lambda^{k}\,\beta x_{ki})^{\gamma}}\right)}\end{bmatrix} \tag{10}$$

Therefore, the maximum likelihood estimates of λ,α andβ are obtained by setting Equations (8), (9) and (10) to be equal zero. Obviously, it is very difficult to obtain a closed form solution for the three non-linear equations. So Newton- Raphson method is used to get the maximum likelihood estimators of $\lambda,\alpha\ and\beta$

Concerning the asymptotic variance-covariance matrix of the ML estimators of the parameters, it can be approximated by numerically inverting the Fisher-information matrix F. It is composed of the negative second derivatives of the natural logarithm of the likelihood function evaluated at the ML estimates. Therefore, the asymptotic Fisher-information matrix can be written as follows:

$$F=\begin{bmatrix} \frac{-\partial^2 lnL}{\partial\lambda^2} & \frac{-\partial^2 lnL}{\partial\lambda\partial\alpha} & \frac{-\partial^2 lnL}{\partial\lambda\partial\beta} \\ \frac{-\partial^2 lnL}{\partial\lambda\partial\alpha} & \frac{-\partial^2 lnL}{\partial\alpha^2} & \frac{-\partial^2 lnL}{\partial\alpha\partial\beta} \\ \frac{-\partial^2 lnL}{\partial\lambda\partial\beta} & \frac{-\partial^2 lnL}{\partial\alpha\partial\beta} & \frac{-\partial^2 lnL}{\partial\beta^2} \end{bmatrix}\downarrow(\lambda=\hat{\lambda}\,,\alpha=\hat{\alpha}\,,\beta=\hat{\beta}\ ) \tag{11}$$

The elements of Fisher information matrix F can be expressed by the following equations:

$$\frac{\partial^2 lnL}{\partial\lambda^2}=\sum_{k=1}^{s}\sum_{i=1}^{n}\begin{bmatrix} \frac{-k\gamma}{\lambda^2}-k\gamma(k\gamma-1)\lambda^{k\gamma-2}(\ \beta x_{ki})^{\gamma} \\ +\frac{(\alpha-1)e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}k\gamma\lambda^{k\gamma-2}(\ \beta x_{ki})^{\gamma}\psi}{\left(1-e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}\right)^2} \end{bmatrix} \tag{12}$$

$$\psi=(k\gamma-1)\left(1-e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}\right)-k\gamma(\ \beta x_{ki})^{\gamma}\lambda^{k\gamma}$$

$$\frac{\partial^2 lnL}{\partial\lambda\partial\alpha}=\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{k\gamma\lambda^{k\gamma-1}(\ \beta x_{ki})^{\gamma}e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}}{\left(1-e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}\right)}\right] \tag{13}$$

$$\frac{\partial^2 lnL}{\partial\lambda\partial\beta}=\sum_{k=1}^{s}\sum_{i=1}^{n}\begin{bmatrix} -k\gamma^2\beta^{\gamma-1}\lambda^{k\gamma-1}x_{ki}^{\gamma} \\ +\frac{(\alpha-1)k\gamma^2x_{ki}^{\gamma}\lambda^{k\gamma-1}\beta^{\gamma-1}e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}\phi}{\left(1-e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}\right)^2} \end{bmatrix} \tag{14}$$

$$\phi=1-\beta^{\gamma}\lambda^{k\gamma}x_{ki}^{\gamma}-e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}$$

$$\frac{\partial^2 lnL}{\partial\alpha^2}=\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{-1}{\alpha^2}\right] \tag{15}$$

$$\frac{\partial^2 lnL}{\partial\alpha\partial\beta}=\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{\gamma\beta^{\gamma-1}\lambda^{k\gamma}x_{ki}^{\gamma}e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}}{\left(1-e^{-(\lambda^k\,\beta x_{ki})^{\gamma}}\right)}\right] \tag{16}$$

$$\frac{\partial^2 lnL}{\partial\beta^2}=\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\begin{array}{c}\frac{-\gamma}{\beta^2}-\gamma(\gamma-1)\beta^{\gamma-2}\lambda^{k\gamma}x_{ki}^{\gamma}\\ +\frac{(\alpha-1)e^{-(\lambda^k\beta x_{ki})^{\gamma}}\gamma\lambda^{k\gamma}\beta^{\gamma-2}x_{ki}^{\gamma}\omega}{\left(1-e^{-(\lambda^k\beta x_{ki})^{\gamma}}\right)^2}\end{array}\right] \tag{17}$$

$$\omega=(\gamma-1)\left(1-e^{-(\lambda^k\beta x_{ki})^{\gamma}}\right)-\gamma\lambda^{k\gamma}\beta^{\gamma}x_{ki}^{\gamma}$$

4. Confidence Interval for parameters Estimation

4.1 Asymptotic Confidence Interval

an asymptotic variance covariance matrix $V=F^{-1}$ defined by inverting the Fisher information matrix F and substituting $\hat{\lambda}\ for\ \lambda\ ,\hat{\alpha}\ \ for\ \alpha\ and\ \hat{\beta}\ \ for\ \beta$.
The 100 $(1-\delta)$% asymptotic confidence intervals for $\lambda\ ,\alpha\ and\ \ \beta$ are then given respectively by:

$$\left[\hat{\lambda}\ \pm Z_{1-\frac{\delta}{2}}\sqrt{var(\hat{\lambda})}\right],\ \left[\hat{\alpha}\ \pm Z_{1-\frac{\delta}{2}}\sqrt{var(\hat{\alpha})}\right]\ \text{and}\ \left[\hat{\beta}\ \pm Z_{1-\frac{\delta}{2}}\sqrt{var(\hat{\beta})}\right]$$

4.2 Bootstrap Confidence Interval

We construct the confidence intervals for $\lambda\ ,\alpha\ and\ \ \beta$ based on the parametric bootstrap, using the percentile bootstrap interval method. We describe the algorithm to obtain the coverage for bootstrap CI as below:

1. Use the samples for $k=1,2,\dots,s$,and $i=1,2,\dots,n$ which indicated to life times x_{ki} and equations (8), (9) and (10) to compute the estimates of maximum likelihood $\hat{\lambda}\ ,\hat{\alpha}\ \ and\ \hat{\beta}$.
2. Use the estimates $\hat{\lambda}\ ,\hat{\alpha}\ \ and\ \hat{\beta}$ to generate a bootstrap sample x_{ki}.
3. Repeat Step 2, 1000 BOOT times.
4. we can get three vectors of MLs for each parameter which contains 1000 observations.

Then the approximate $100(1-\delta)$)% confidence intervals for $\lambda\ ,\alpha\ and\ \ \beta$ are given by arranging the values in ascending order to obtain the confidence intervals as: $\left[\hat{\lambda}^*(\delta),\hat{\lambda}^*(1-\delta)\right]$, $[\hat{\alpha}^*(\delta),\hat{\alpha}^*(1-\delta)]$ and $\left[\hat{\beta}^*(\delta),\hat{\beta}^*(1-\delta)\right]$.

5. Periodical Preventive Maintenance Schedule

Let the reliability of a system without maintenance be given by $R(x)$ and let the preventive maintenance be carried out periodically at times $t_0, 2t_0, 3t_0, \ldots, nt_0$. It is assumed that the system restores to "as good as new" after every maintenance action. Let $\boldsymbol{T}$ represent the total operating time for the system. Then the reliability $R(m)$ of the maintained system will be

$R(m) = R^n(\boldsymbol{t_0})\, R(T - nt_0) \qquad nt_0 \leq T \leq (n+1)t_0\, , n = 0,1,2, \ldots, k$

Preventive maintenance improves the reliability of a system during the wear out period. As Chitra (2003), This strategy is also known as age replacement. The cost of improving the reliability through preventive maintenance should be compared with the cost of restoring a system through repair. Therefore, the total cost C involved in the replacement of a particular component is given by

$$C = \frac{T}{D} \; [(C_P - C_r)\, R(t_0) + C_r] \tag{18}$$

$$D = \int_0^{t_0} R(\tau)\; d\tau$$

Where $R(\tau)$ is the reliability of the original component which is automatically replaced after time t_0 .
The value of t_0 corresponding to the minimum cost is given by differentiating equation ( ) with respect to t_0:

$$\frac{dC}{d\, t_0} = \frac{T}{D^2} \; \left[D\, (C_P - C_r)\, R^{\backslash}(t_0) - (C_P - C_r)\, R^2(t_0) - C_r\, R(t_0)\right] \tag{19}$$

By equating the equation (19) to zero and being nonlinear can be solved numerically for obtaining *to*, therefore, we obtained the second derivative of equation (18) with respect to *to*:

$$\frac{d^2C}{d\, {t_0}^2} = \frac{T}{D^3} \; \left[D^2(C_P - C_r)\, R^{\backslash\backslash}(t_0) - 3D^2(C_P - C_r)\, R(t_0)R^{\backslash}(t_0) - DC_r\, R^{\backslash}(t_0) + 2C_P R^3(t_0)\right] \tag{20}$$

where

$$R(t_0) = 1 - (1 - e^{-(\hat{\beta} t_0)^{\gamma}})^{\hat{\alpha}} \tag{21}$$

$$R^{\backslash}(t_0) = -\hat{\alpha}\gamma\hat{\beta}^{\gamma} {t_0}^{\gamma-1} (1 - e^{-(\hat{\beta} t_0)^{\gamma}})^{\hat{\alpha}-1} \, e^{-(\hat{\beta} t_0)^{\gamma}} \tag{22}$$

$$\begin{aligned} R^{\backslash\backslash}(t_0) = &-\hat{\alpha}\gamma\hat{\beta}^{\gamma} {t_0}^{\gamma-2} \left(1 - e^{-(\hat{\beta} t_0)^{\gamma}}\right)^{\alpha-2} \; e^{-(\hat{\beta} t_0)^{\gamma}} \\ &\left[(\gamma - 1)\left(1 - e^{-(\beta t_0)^{\gamma}}\right) + \gamma\hat{\beta}^{\gamma} {t_0}^{\gamma} (\hat{\alpha} e^{-(\hat{\beta} t_0)^{\gamma}} - 1)\right] \end{aligned} \tag{23}$$

Then we can get the Minimum expected maintenance cost by substituting in equation (18) with the value of periodical preventive maintenance t_0, and expected maintenance cost rate $E(c(t_0, T)) = E(c|t_0)/T$.

6. Numerical study

A simulation study is carried out to investigate the performance of the estimators for items having Exponentiated Weibull distribution based on complete data. The performance of estimators has been considered in terms of their relative absolute bias (RAB) and means square error (MSE).

A simulation study is performed according to the following steps:

Step 1: generate a samples of data having Exponentiated Weibull distribution x_{ik} ; i=1,2,…,n and k=1,2,…,S. at chosen values of the parameters $\alpha, \beta \; and \; \lambda$.

Step 2 : our experiment is done under complete sample which means the experiment terminate when all items are failure.

Step 3: Generate1000 random samples random samples of sizes size $n = 20, 50\,, 100 \; and \; 200$ are generated from Exponentiated Weibull distribution at $\gamma = 2 \; and \; S = 10$. The parameters' values are chosen as:

Case 1: $\alpha = 3, \beta = 0.05 \; and \; \lambda = 2$
Case 2: $\alpha = 3, \beta = 0.06 \; and \; \lambda = 2$
Case 3: $\alpha = 3.5, \beta = 0.06 \; and \; \lambda = 2$
Case 4: $\alpha = 3.5, \beta = 0.06 \; and \; \lambda = 3$

Step 4: For each sample and for the selected sets of parameters, the distribution parameters $\alpha$and β and the acceleration factor λ are estimated. Newton -Raphson technique is applied for solving the nonlinear Equations (8), (9) and (10) to get the estimates of α, β and λ .

Step 5: The Relative Bias and MSE of the estimators for the distribution parameters and acceleration factor for all sample sizes are computed.

Step 6: The asymptotic variance and covariance matrix of the estimators for different sample sizes are obtained by using Equations (11-17).

Step 7: Using normal approximation method to construct the two sided confidence limits with confidence levels and of the acceleration factor and the two parameters are constructed $\delta = 95\%$

Step8: Using Bootstrap method to construct the two sided confidence limits for parameters α, β and λ at $\delta = 95\%$

Step 9: using the estimated parameters and confidence limits to predict Periodical Preventive Maintenance Schedule t_0 as in equations (18-23) at $T = 5000$, $C_P = 100$, $C_r = 1000$.

Our simulation results are summarized in tables 1 to 4:

Tables 1 and 2 give the maximum likelihood estimators (MLE) for Exponentiated Weibull distribution parameters and The accelerated factor, Means square error (MSE) , Relative Absolute bias (RAB) and asymptotic variance and covariance matrix of estimators are calculated for the selected set of parameters and different sample sizes.

. In table 3, Confidence interval for model parameters are constructed at confidence level 95% using normal approximation and percentile Bootstrap methods. Table 4 shows Periodical Preventive Maintenance Schedule , the Minimum maintenance cost and the expected maintenance cost rate with Confidence Interval at 95%.

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Table (1)
The MLE, Relative Absolute Bias and MSE of the Estimators

n	parameter	$\alpha = 3, \beta = 0.05$ *and* $\lambda = 2$			$\alpha = 3, \beta = 0.06$ *and* $\lambda = 2$		
		MLE	MSE	RAB	MLE	MSE	RAB
20	α	2.990414	0.172643	0.003195	2.990039	0.1730028	0.00332
	β	0.050117	4.1113E-6	0.002340	0.0601419	5.9269E-6	0.002366
	λ	1.998182	3.5647E-4	0.000909	1.998159	3.577E-4	0.0009207
50	α	2.996676	0.06900896	0.0011078	2.997226	0.069492	9.245E-4
	β	0.0500592	1.733E-6	0.0011840	0.0600708	2.500E-6	0.001811
	λ	1.999115	1.490E-4	0.0004425	1.99913	1.496E-4	0.0004349
100	α	2.996773	0.0339808	0.0010758	2.99753	0.034478	0.0008233
	β	0.0499997	8.395E-7	4.395E-6	0.0600014	1.213E-6	0.0000238
	λ	1.999709	7.139E-5	1.456E-4	1.999718	7.245E-5	0.0001409
200	α	2.997435	0.014955	0.000854	2.997389	0.015077	8.703E-4
	β	0.0500169	4.052E-7	0.000338	0.0600200	5.852E-7	3.335E-4
	λ	1.99975	3.462E-5	0.000124	1.99975	3.480E-5	1.25E-4
		$\alpha = 3.5, \beta = 0.06$ *and* $\lambda = 2$			$\alpha = 3.5, \beta = 0.06$ *and* $\lambda = 3$		
20	α	3.489896	0.1440921	0.002886	3.489887	0.144482	0.002889
	β	0.060086	4.304E-6	0.001434	0.0600603	4.309E-6	0.0014338
	λ	1.998991	1.737E-4	0.000504	2.998486	3.91 E-4	0.000504
50	α	3.498963	0.054972	0.000296	3.498945	0.055375	0.000301
	β	0.060046	0.000768	0.000768	0.0600455	1.77E-6	0.000759
	λ	1.999516	0.000241	0.000241	2.99928	0.000166	0.00024
100	α	3.496062	0.0277039	0.001125	3.496084	0.027907	0.001118
	β	0.0599945	8.73E-7	9.139E-5	0.0599945	8.75E-7	9.05E-5
	λ	1.999828	3.56E-5	8.594E-5	2.999743	8.026E-5	8.026E-5
200	α	3.497528	0.012089	0.000706	3.497488	0.012139	0.000717
	β	0.060014	4.097E-7	2.477E-4	0.0600143	4.096E-7	0.000239
	λ	1.999833	1.712E-5	8.35E-5	2.999753	3.847E-5	0.000082

Table (2)
Asymptotic Variance and Covariance Matrix of Estimators

		$\alpha = 3, \beta = 0.05$ and $\lambda = 2$			$\alpha = 3, \beta = 0.06$ and $\lambda = 2$		
		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$
20	$\hat{\alpha}$	3.2756E-6	-2.220E-6	5.397E-4	3.275E-6	-2.665E-6	5.397E-4
	$\hat{\beta}$		1.124E-3	-7.11E-5		1.124E-3	-8.536E-5
	$\hat{\lambda}$			2.187E-6			3.15E-6
50	$\hat{\alpha}$	1.126E-6	-7.963E-7	2.094E-4	1.126E-6	-9.555E-7	2.094E-4
	$\hat{\beta}$		4.424E-4	-2.73E-5		4.424E-4	-3.287E-5
	$\hat{\lambda}$			7.97E-7			1.147E-6
100	$\hat{\alpha}$	4.457E-7	-3.645E-7	1.012E-4	4.457E-7	-4.37E-7	1.012E-4
	$\hat{\beta}$		2.053E-4	-1.22E-5		2.053E-4	-1.533E-5
	$\hat{\lambda}$			3.474E-7			5.003E-7
200	$\hat{\alpha}$	1.983E-7	-1.778E-7	4.962E-5	1.983E-7	-2.134E-7	4.962E-5
	$\hat{\beta}$		1.008E-4	-6.47E-6		1.008E-4	-7.77E-6
	$\hat{\lambda}$			1.77E-7			2.558E-7
		$\alpha = 3.5, \beta = 0.06$ and $\lambda = 2$			$\alpha = 3.5, \beta = 0.06$ and $\lambda = 3$		
20	$\hat{\alpha}$	1.787E-7	-2.045E-6	4.94E-4	4.02E-6	-3.06E-6	7.41E-4
	$\hat{\beta}$		0.00108	-7.90E-5		0.00108	-7.905E-5
	$\hat{\lambda}$			2.60E-6			2.605E-6
50	$\hat{\alpha}$	5.306E-7	-6.665E-7	1.87E-4	1.193E-6	-9.99E-7	2.805E-4
	$\hat{\beta}$		4.06E-4	-2.90E-5		4.069E-4	-2.905E-5
	$\hat{\lambda}$			8.59E-7			8.597E-7
100	$\hat{\alpha}$	2.604E-7	-3.45E-7	9.39E-5	5.86E-7	-5.183E-7	1.408E-4
	$\hat{\beta}$		2.002E-4	-1.43E-5		2.002E-4	-1.43E-5
	$\hat{\lambda}$			4.27E-7			4.27E-7
200	$\hat{\alpha}$	1.19E-7	-1.695E-7	4.621E-5	2.677E-7	-2.543E-7	6.932E-7
	$\hat{\beta}$		9.861E-5	-7.28E-6		9.86E-5	-7.289E-6
	$\hat{\lambda}$			2.194E-7			2.194E-7

Table (3)
Confidence Interval for Model Parameters at Confidence Level 95%
Using Normal Approximation and Percentile Bootstrap Methods

		$\alpha = 3, \beta = 0.05$ *and* $\lambda = 2$				$\alpha = 3, \beta = 0.06$ *and* $\lambda = 2$			
		Using Normal Approximation		Using Percentile Bootstrap		Using Normal Approximation		Using Percentile Bootstrap	
n	parameter	lower	upper	lower	upper	lower	upper	lower	upper
	α	2.98686	2.99396	2.28084	3.67891	2.986491	2.993586	2.28084	3.678912
	β	0.047218	0.05301	0.04679	0.05350	0.056663	0.063621	0.056152	0.064209
20	λ	1.932444	2.06392	1.96637	2.026961	1.932421	2.063896	1.966378	2.026961
	α	2.994596	2.99875	2.55773	3.426304	2.995147	2.999306	2.557731	3.426304
	β	0.048309	0.05180	0.04791	0.052216	0.057971	0.062170	0.05749	0.062659
50	λ	1.957889	2.04034	1.97798	2.019041	1.957904	2.040357	1.977984	2.019041
	α	2.995464	2.99808	2.69518	3.303881	2.996221	2.998839	2.695181	3.305717
	β	0.048844	0.05115	0.04852	0.051489	0.058614	0.061387	0.058232	0.061787
100	λ	1.971619	2.02779	1.98539	2.01313	1.971628	2.027808	1.985314	2.013148
	α	2.996562	2.99830	2.78950	3.205471	2.996516	2.998262	2.789504	3.205471
	β	0.049190	0.05084	0.04897	0.051120	0.059028	0.061011	0.058771	0.061344
200	λ	1.980065	2.01943	1.99008	2.009876	1.980065	2.019435	1.990083	2.009876
		$\alpha = 3.5, \beta = 0.06$ *and* $\lambda = 2$				$\alpha = 3.5, \beta = 0.06$ *and* $\lambda = 3$			
	α	3.487276	3.492517	2.85656	4.120306	3.485956	3.493818	2.856567	4.120308
	β	0.056922	0.06324	0.056586	0.063475	0.056922	0.063249	0.056586	0.063475
20	λ	1.934533	2.06344	1.977454	2.019907	2.934028	3.062944	2.966182	3.029861
	α	3.497535	3.500391	3.11403	3.887137	3.496803	3.501086	3.11403	3.887135
	β	0.058228	0.061863	0.057817	0.062278	0.058228	0.061862	0.057817	0.062278
50	λ	1.959978	2.039055	1.984643	2.014001	2.95974	3.038819	2.976964	3.021019
	α	3.495061	3.497062	3.221589	3.775717	3.494584	3.497585	3.218381	3.775713
	β	0.058712	0.061276	0.058426	0.061515	0.058712	0.0612767	0.058426	0.061515
100	λ	1.972094	2.027562	1.989669	2.009656	2.972009	3.027477	2.984504	3.014484
	α	3.496852	3.498204	3.309151	3.684491	3.496473	3.498502	3.309149	3.684493
	β	0.059096	0.060933	0.058979	0.061130	0.059096	0.0609325	0.058979	0.061130
200	λ	1.980369	2.019297	1.99303	2.00714	2.980289	3.019217	2.989545	3.010711

Table (4)
Periodical Preventive Maintenance Schedule , The Minimum maintenance cost and the Expected Maintenance cost rate with Confidence Interval at 95%

	n	20	50	100	200
		$\alpha = 3, \beta = 0.05$ *and* $\lambda = 2$			
Point Estimation	t_0	32.9949	33.2477	33.27916	33.27164
	$E(C\|t_0)$	696618.6	701960	700889.3	701228.1
	$E(C(t_0,T)$	139.3237	140.392	140.1779	140.2456
Normal Approximation Interval 95%	t_0	(31.366,35.215)	(32.159, 34.399)	(32.4826, 33.896)	(32.604,33.710)
	$E(C\|t_0)$	(661478.8, 742599.1)	(676040.2, 727538)	(679971, 715683)	(686263,710058)
	$E(C(t_0,T)$	(132.295, 148.519)	(135.208, 145.507)	(135.994,143.136)	(137.25, 142.01)
Percentile Bootstrap Interval 95%	t_0	(32.612, 32.751)	(33.008, 33.263)	(33.149, 33.316)	(33.143, 33.323)
	$E(C\|t_0)$	(604186.9, 786227.8)	(643648.6, 758951)	(661039, 740740)	(673356, 730009)
	$E(C(t_0,T)$	(120.837, 157.245)	(128.729, 151.790)	(132.207, 148.14)	(134.67, 146.001)
	$\alpha = 3, \beta = 0.06$ *and* $\lambda = 2$				
Point Estimation	t_0	27.59939	27.63881	27.6646	27.65761
	$E(C\|t_0)$	840393.3	839480	838241.6	838570
	$E(C(t_0,T)$	168.0786	167.896	167.6483	167.714
Normal Approximation Interval 95%	t_0	(25.99829,29.3446)	(26.6978, 28.632)	(27.171,28.386)	(27.303,28.229)
	$E(C\|t_0)$	(793784.6,884534.9)	(809917, 868424.6)	(821606, 863385)	(829103, 856540)
	$E(C(t_0,T)$	(158.7569,176.907)	(161.983, 173.684)	(164.321,172.677)	(165.82, 171.30)
Percentile Bootstrap Interval 95%	t_0	(27.26976,27.41366)	(27.4436, 27.6444)	(27.562, 27.691)	(27.553, 27.697)
	$E(C\|t_0)$	(729310.7, 948053.3)	(769544.4,907785)	(790394, 886038)	(805162, 873083)
	$E(C(t_0,T)$	(145.8621, 189.6107)	(153.908, 181.557)	(158.078,177.207)	(161.03, 174.61)
	$\alpha = 3.5, \beta = 0.06$ *and* $\lambda = 2$				
Point Estimation	t_0	28.68137	28.71093	28.72628	28.72073
	$E(C\|t_0)$	871669.9	871327	870244	870685
	$E(C(t_0,T)$	174.334	174.2654	174.049	174.137
Normal Approximation Interval 95%	t_0	(27.336,30.345)	(28.030, 29.622)	(28.242, 29.422)	(28.372, 29.270)
	$E(C\|t_0)$	(828531,921779)	(845633, 905105)	(854489, 894056)	(861694, 887730)
	$E(C(t_0,T)$	(165.706, 184.356)	(169.126, 181.021)	(170.898, 178.81)	(172.33, 177.546)
Percentile Bootstrap Interval 95%	t_0	(28.276, 28.895)	(28.423, 28.921)	(28.558, 28.878)	(28.575, 28.814)
	$E(C\|t_0)$	(778051, 961187)	(812921, 929315)	(829059, 910437)	(843209, 899442)
	$E(C(t_0,T)$	(155.610, 192.237)	(162.584, 185.863)	(165.811, 182.08)	(168.64, 179.88)

6- CONCLUSIONS

From tables (1-4), the following observations can be made on the performance of estimated parameters of Exponentiated Weibull distribution based on Geometric Process Model for the Analysis of Accelerated Life Testing with Complete Data:

It is observed that The MSE and RAB of α decreases as the increasing the values of α *and* n but increases with the increasing values of β *and* λ. There are the smallest values of MSE and RAB for β *and* λ which decreases as n increase. The asymptotic variances of the estimates as in table (2) decrease as the sample size increases, therefore the confidence interval length for the estimated parameters in table (3) whether in normal approximation or Bootstrap method decreases. In general, the length of interval for Bootstrap method is greater than it in normal approximation. The Periodical Preventive Maintenance Schedule is increasing slowly as n increasing. There is inverse relationship between the values of β and The Periodical Preventive Maintenance Schedule t_0 while it is directly with the expected minimum maintenance cost and the expected maintenance cost rate. The value of α effect directly in all measures $t_0, E(C|t_0)$ *and* $E(C(t_0, T)$.

References

[1] A Siswanto , N Kurniati, (2017), " Determining optimal preventive maintenance interval for component of Well Barrier Element in an Oil & Gas Company", International Conference on Industrial and System Engineering (IConISE) , pp. 1-7.

[2] A. Chitra, (2003), " Life Based Maintenance Policy for Minimum Cost", 2003 Proceedings Annual Reliability and Maintainability Symposium, pp. 470-474.

[3] Amit Kumar Jain, (2012), " An Optimal Preventive Maintenance Strategy for Efficient Operation of Boilers in Industry", Control Theory and Informatics, Vol 2, No.4, pp.1-3.

[4] B.P. Iskandara and H. Husniah , (2017), " Optimal preventive maintenance for a two dimensional lease contract", Computers & Industrial Engineering , v. 113, pp.693 – 703.

[5] Dmitriy Belyi , Elmira Popova , David P. Morton , Paul Damien, (2017), " Bayesian failure-rate modeling and preventive maintenance optimization", European Journal of Operational Research, v. 262 , pp. 1085–1093.

[6] Fabio Sgarbossa, Ilenia Zennaro, Eleonora Florian, Alessandro Persona, (2018), " Impacts of weibull parameters estimation on preventive maintenance cost", IFAC (International Federation of Automatic Control) Papers OnLine, v. 51-11 , pp. 508–513 .

[7] G.S. Mudholkar, D.K. Srivastava (1993), " Exponentiated Weibull family for analyzing bathtub failure-rate data" , IEEE. Transaction on Reliability , 42 , pp. 299-302

[8] Huang, S., “Statistical inference in accelerated life testing with geometric process model”, Master’s thesis, San Diego State University, (2011).

[9] Kamal Ullah, Inthekhab Alam, Showkat Ahmad Lone, (2017), " Accelerated Life Testing Design Using Geometric Process for Generalized Rayleigh Distribution with Complete Data", RT&A, No 4 (47), pp. 76-90.

[10] Kamran S. Moghaddam and John S. Usher, (2010), "Optimal preventive maintenance and replacement schedules with variable improvement factor", Journal of Quality in Maintenance Engineering, Vol. 16 No. 3, 2010, pp. 271-287.

[11] Lianfen Qian, (2012), "The Fisher information matrix for a three-parameter exponentiated Weibull distribution under type II censoring", Statistical Methodology, Volume 9, Issue 3, pp. 320-329.
[12] M. Kamal, (2013), " Application of Geometric Process in accelerated life testing analysis with Type –I censored Weibull failure data", Reliability: Theory & Applications, Vol. 8, No. 3, pp. 87-96

[13] M. Kamal, S. Zarrin, S. Saxena and A. Islam, “Weibull Geometric Process Model for the Analysis of Accelerated Life Testing with Complete Data”, International Journal of Statistics and Applications, vol. 2, no. 5, pp. 60-66,2012.
[14] M. Kamal, S. Zarrin, S. Saxena, and A.Islam, “Accelerated life test design using geometric process for Pareto distribution”, vol.1, no.2, pp. 25-31, 2013.
[15] M. M. Mohie El-Din , S. E. Abu-Youssef , Nahed S. A. Ali and A. M. Abd El-Raheem , (2018)," Inference on Constant-Stress Accelerated Life Testing Based on Geometric Process for Extension of the Exponential Distribution under Type-II Progressive Censoring. Pak.j.stat.oper.res. Vol.XIV, No.2, pp 245-263.
[16] Mashail M. AL Sobhi, Ahmed A. Soliman, (2016), " Estimation for the exponentiated Weibull model with adaptive Type-II progressive censored schemes", Applied Mathematical Modelling, Volume 40, Issue 2, Pages 1180-1192.
[17] Minjae Park , Ki Mun Jung , Dong Ho Park, (2018), " Optimization of periodic preventive maintenance policy following the expiration of two-dimensional warranty", Reliability Engineering and System Safety, v. 170 , pp. 1–9.
[18] R. Jiang ; D.N.P. Murthy , (1999), "The exponentiated Weibull family: a graphical approach ", IEEE Transactions on Reliability ,Volume: 48 , Issue 1 , pp. 68 – 72.
[19] Sadia Anwar, Sana Shahab, Arif Ul Islam, (2014), "Accelerated Life Testing Design Using Geometric Process For Marshall-Olkin Extended Exponential Distribution With Type I Censored Data", International Journal of Scientific & Technology Research, vol. 3, Issue 1 , pp. 179-186.
[20] Sana Shahab, Arif-Ul-Islam, (2013), " Analysis of Accelerated Life Testing using Geometric Process for Frechiet distribution ", International Journal of Engineering Sciences & Research Technology, Vol. 2, Issue 9,pp. 4987-4995.

[21] Sana Shahab, Arif-Ul-Islam, (2013), " Constant Accelerated Life Testing with Geometric Process for Marshall-Olkin Lomax Distribution", International Journal of Scientific & Engineering Research, Volume 4, Issue 12,pp. 1914-1920.

[22] Saxena, S. and Islam, A.,(2013), " Constant stress accelerated life testing using Rayleigh geometric process model", International Journal of Engineering Sciences & Research Technology, 2(9).

[23] Showkat Ahmad Lone, Ahmadur Rahman, Arif-Ul-Islam, (2016), " A Study of accelerated life testing design using geometric process for generalized exponential distribution using time constraint" , International Journal of Engineering Sciences & Research Technology, 5(2), pp. 91-100.

[24] U. Singh, P.K. Gupta, S.K. Upadhyay, (2005) " Estimation of three-parameter exponentiated-Weibull distribution under type-II censoring", Journal of Statistic Planning Inference, v. 134, pp. 350–372.